Synthetic Division

To find a quotient as well as the remainder when a polynomial is divided by $x-c$, a shortened version of long division, called synthetic division, makes the task simpler.

Steps:

1. Write the constant c of the divisor $x - c$ in a box. **(Opposite sign!)**
2. Write the coefficients of the dividend (including 0's for any missing powers of x) in descending order to the right of the box.
3. Bring the leading coefficient down two rows and enter it into row 3.
4. Multiply the latest entry in row 3 by c, then write the answer in row 2 below the next coefficient in the dividend (one column to the right).
5. Add the entry just written in row 2 to the coefficient above it in row 1. Record the sum in row 3.
6. Repeat steps 4 and 5 until no more entries remain in row 1.
7. The numbers in row 3 are the coefficients of the quotient, with the final number at the right being the remainder.

Example: Solve $\left(2x^3 - x^2 + 3\right) \div (x-3)$ using both long division and synthetic division.

Long Division

$$
\begin{array}{r}
2x^2 + 5x + 15 \\
x-3 \;\big)\; 2x^3 - x^2 + 0x + 3 \\
-(2x^3 - 6x^2) \qquad\qquad \\
\hline
5x^2 + 0x \qquad \\
-(5x^2 - 15x) \qquad \\
\hline
15x + 3 \\
-(15x - 45) \\
\hline
48
\end{array}
$$

Synthetic Division

$$
\begin{array}{r|rrrr}
\underline{3} & 2 & -1 & 0 & 3 \\
 & & 6 & 15 & 45 \\
\hline
 & 2 & 5 & 15 & \vert\, 48 \leftarrow \text{remainder}
\end{array}
$$

$$\boxed{2x^2 + 5x + 15 + \frac{48}{x-3}}$$

Example: Use synthetic division to find the quotient and remainder for the following problem:
$\left(-4x^3 + 2x^2 - x + 1\right) \div (x+2)$

$$
\begin{array}{r|rrrr}
\underline{-2} & -4 & 2 & -1 & 1 \\
 & & 8 & -21 & 42 \\
\hline
 & -4 & 10 & -21 & \vert\, 43
\end{array}
$$

$$\boxed{-4x^2 + 10x - 21 \quad R43}$$

Example: Use synthetic division to show that $x-9$ is a factor of $2x^3 - 18x^2 + x - 9$.

If something is a factor, that means it divides evenly with a remainder of zero.

$$
\begin{array}{r|rrrr}
\underline{9} & 2 & -18 & 1 & -9 \\
 & & 18 & 0 & 9 \\
\hline
 & 2 & 0 & 1 & \vert\, 0 \leftarrow
\end{array}
$$

Remainder 0, so $x-9$ is a factor